

General Certificate of Education
January 2009
Advanced Level Examination



MATHEMATICS
Unit Pure Core 4

MPC4

Wednesday 21 January 2009 1.30 pm to 3.00 pm

For this paper you must have:

- an 8-page answer book
 - the blue AQA booklet of formulae and statistical tables.
- You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MPC4.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.

Information

- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

- 1 (a) The polynomial $f(x)$ is defined by $f(x) = 4x^3 - 7x - 3$.
- (i) Find $f(-1)$. (1 mark)
- (ii) Use the Factor Theorem to show that $2x + 1$ is a factor of $f(x)$. (2 marks)
- (iii) Simplify the algebraic fraction $\frac{4x^3 - 7x - 3}{2x^2 + 3x + 1}$. (3 marks)
- (b) The polynomial $g(x)$ is defined by $g(x) = 4x^3 - 7x + d$. When $g(x)$ is divided by $2x + 1$, the remainder is 2. Find the value of d . (2 marks)
- 2 (a) Express $\sin x - 3 \cos x$ in the form $R \sin(x - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. Give your value of α in radians to two decimal places. (3 marks)
- (b) Hence:
- (i) write down the minimum value of $\sin x - 3 \cos x$; (1 mark)
- (ii) find the value of x in the interval $0 < x < 2\pi$ at which this minimum value occurs, giving your value of x in radians to two decimal places. (2 marks)
- 3 (a) (i) Express $\frac{2x + 7}{x + 2}$ in the form $A + \frac{B}{x + 2}$, where A and B are integers. (2 marks)
- (ii) Hence find $\int \frac{2x + 7}{x + 2} dx$. (2 marks)
- (b) (i) Express $\frac{28 + 4x^2}{(1 + 3x)(5 - x)^2}$ in the form $\frac{P}{1 + 3x} + \frac{Q}{5 - x} + \frac{R}{(5 - x)^2}$, where P , Q and R are constants. (5 marks)
- (ii) Hence find $\int \frac{28 + 4x^2}{(1 + 3x)(5 - x)^2} dx$. (4 marks)

- 4 (a) (i) Find the binomial expansion of $(1 - x)^{\frac{1}{2}}$ up to and including the term in x^2 .
(2 marks)
- (ii) Hence obtain the binomial expansion of $\sqrt{4 - x}$ up to and including the term in x^2 .
(3 marks)
- (b) Use your answer to part (a)(ii) to find an approximate value for $\sqrt{3}$. Give your answer to three decimal places.
(2 marks)

- 5 (a) Express $\sin 2x$ in terms of $\sin x$ and $\cos x$.
(1 mark)
- (b) Solve the equation

$$5 \sin 2x + 3 \cos x = 0$$

giving all solutions in the interval $0^\circ \leq x \leq 360^\circ$ to the nearest 0.1° , where appropriate.
(4 marks)

- (c) Given that $\sin 2x + \cos 2x = 1 + \sin x$ and $\sin x \neq 0$, show that $2(\cos x - \sin x) = 1$.
(4 marks)

6 A curve is defined by the equation $x^2y + y^3 = 2x + 1$.

- (a) Find the gradient of the curve at the point $(2, 1)$.
(6 marks)
- (b) Show that the x -coordinate of any stationary point on this curve satisfies the equation

$$\frac{1}{x^3} = x + 1 \quad (4 \text{ marks})$$

Turn over for the next question

Turn over ►

- 7 (a) A differential equation is given by $\frac{dx}{dt} = -kte^{\frac{1}{2}x}$, where k is a positive constant.
- (i) Solve the differential equation. (3 marks)
- (ii) Hence, given that $x = 6$ when $t = 0$, show that $x = -2 \ln\left(\frac{kt^2}{4} + e^{-3}\right)$. (3 marks)
- (b) The population of a colony of insects is decreasing according to the model $\frac{dx}{dt} = -kte^{\frac{1}{2}x}$, where x thousands is the number of insects in the colony after time t minutes. Initially, there were 6000 insects in the colony.
- Given that $k = 0.004$, find:
- (i) the population of the colony after 10 minutes, giving your answer to the nearest hundred; (2 marks)
- (ii) the time after which there will be no insects left in the colony, giving your answer to the nearest 0.1 of a minute. (2 marks)
- 8 The points A and B have coordinates $(2, 1, -1)$ and $(3, 1, -2)$ respectively. The angle OBA is θ , where O is the origin.
- (a) (i) Find the vector \overrightarrow{AB} . (2 marks)
- (ii) Show that $\cos \theta = \frac{5}{2\sqrt{7}}$. (4 marks)
- (b) The point C is such that $\overrightarrow{OC} = 2\overrightarrow{OB}$. The line l is parallel to \overrightarrow{AB} and passes through the point C . Find a vector equation of l . (2 marks)
- (c) The point D lies on l such that angle $ODC = 90^\circ$. Find the coordinates of D . (4 marks)

END OF QUESTIONS